VELOCITY OF THE NONSPHERICAL COLLAPSE OF A CAVITATION BUBBLE BETWEEN TWO SOLID WALLS

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The collapse velocity of a cavitation bubble between two solid walls is numerically found on the basis of a nonspherical model for the collapse.

The collapse velocity of a bubble between two solid walls was first calculated in [1], but the spherical model for the collapsing cavitation bubble adopted there is not valid if the maximum initial dimensions of the bubble are comparable to the distance from the center of the bubble to the walls [2]. In other words, this model cannot be used in the case of acoustic cavitation, which is the most common type of cavitation in practical applications [3].

In the present paper we adopt the model of a nonspherical collapse of the cavitation bubble, developed in [2], to determine the collapse velocity.

We assume that the cavitation bubble is initially a sphere and is at the center of the spherical coordinate system $\{r, \theta\}$ in an ideal incompressible liquid which is bounded by two parallel solid walls. The bubble is separated from the two walls by equal distances (Fig. 1). We also assume that the liquid is initially at rest; we ignore gravitation and surface tension; and we assume that the changes in the volume of the vapor–gas mixture in the bubble are adiabatic with $\gamma = 4/3$. The cavitation bubble collapses under the influence of a static pressure.

Under these assumptions, working from the solution of the Laplace equation for the velocity potential for the liquid flow, taking into account the boundary conditions at the solid walls, at the bubble, and at infinity, and taking into account the initial conditions on the radius of the bubble and its collapse velocity, we find that the bubble radius at a given time is expressed by

$$\frac{R(\theta,\ t)}{R_{\max}} = x_0(t) + \sum_{n=1}^{\infty} \lambda^n \left[\sum_{m=0}^{k} x_{n,2m}(t)\, P_{2m}(\cos\theta) \right], \quad 2k < n, \tag{1}$$

where the functions $x_0(t)$ and $x_{n,2m}(t)$ with $n = 1, 2, 3, 4$ and $m = 0, 1$ are found from a stationary system of seven second-order ordinary differential equations [2]:

To find the collapse velocity of a cavitation bubble between two solid walls, we differentiate Eq. (1) with respect to the dimensionless time $\tau = t(p_\infty/\rho)^{1/2}R_{\max}$:

$$\frac{\partial}{\partial\tau}\left(\frac{R}{R_{\max}}\right) = \frac{v}{\sqrt{p_\infty/\rho}} = x_0' + \sum_{n=1}^{\infty} \lambda^n \left[\sum_{m=0}^{k} x_{n,2m}' P_{2m}(\cos\theta) \right], \quad 2k < n. \tag{2}$$

To calculate the derivatives x_0' and $x_{n,2m}'$, we reduce the original system of second-order ordinary differential equations [2] to a system of 14 first-order ordinary differential equations.

The resulting system of equations with the initial conditions $x_0 = 1$, $x_{n,2m} = x_0' = x_{n,2m}' = 0$ at $\tau = 0$ has been integrated numerically on a Minsk-32 computer by the fifth-order Runge–Kutta–Fel'berg method for gas contents $\delta = 0$ and 0.01. The integration step was monitored

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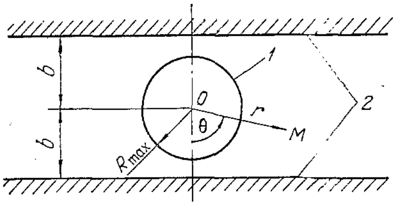

Fig. 1. Initial cavitation bubble, at t = 0. 1) Bubble; 2) solid walls.

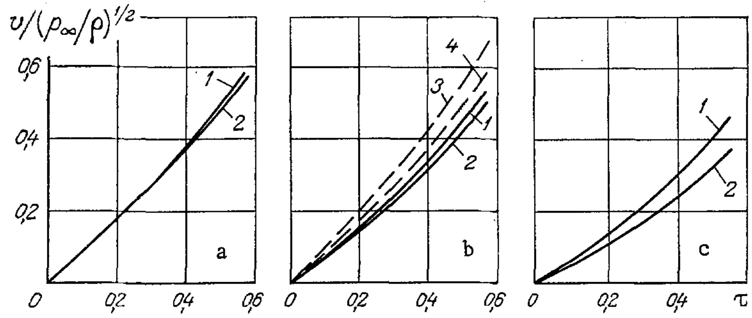

Fig. 2. Change in the dimensionless collapse velocity of a cavitation bubble between two solid walls for δ = 0.01: a) λ = 1/10; b) 1/6; c) 1/3. 1) θ = ±90°; 2, 4) 0°; 3) 180°.

during the calculation. Then the known values of x_0' and $x_{n,2m}'$ were used in Eq. (2) to determine the collapse velocity of the cavitation bubble as a function of the time τ for the values λ = 1/3, 1/6, and 1/10; the angle θ was used as a parameter.

The zero value of δ corresponds to the case in which there is a vacuum in the cavitation bubble, i.e., p_{vg} = 0. In real cavitation bubbles, on the other hand, there is always some amount of gas (air) and vapor of the liquid. Accordingly, δ is not zero, and for a constant liquid temperature it varies over the range $p_v/p_\infty \leqslant \delta \leqslant 1$ [4]. The choice of the value δ = 0.01, which corresponds to a bubble which contains mostly vapor, yields more realistic results on the collapse and permits a comparison with the case of the collapse of a bubble near a single solid wall [5]. The procedure outlined above can also be used for numerical calculations at other possible values of δ.

Figure 2 and Table 1 show the numerical results on the collapse velocity of a cavitation bubble between two solid walls. The dashed curves in Fig. 2b show the corresponding results from [5] for the case of a single solid wall.

It follows from Fig. 2 that the collapse velocity falls off with increasing λ (this decrease is attributed to a degradation of the conditions for the flow of the liquid toward the bubble), and it increases as time elapses.

The collapse velocity is at a maximum at $\theta = \pm\pi/2$, corresponding to the formation of an annular jet. The same conclusion was reached in [2] from a study of the behavior of the shape of the bubble.

It follows from Table 1 that a bubble filled with a vapor–gas mixture collapses more slowly than an empty bubble. The evident reason for this difference is the counterpressure exerted by the vapor–gas mixture as it is compressed. It is not difficult to see that the collapse velocities of empty and filled bubbles differ only slightly, implying that the gas-content parameter δ has only a slight effect on the collapse velocity (for values of this parameter between 0 and 0.01).

Comparison with the results for the case of a single solid wall (Fig. 2b) shows that the second wall reduces the collapse velocity; this conclusion agrees with the results [1], found from a spherical model for the bubble in the case R << b.

TABLE 1. Values of $v/(p_\infty/\rho)^{1/2}$

δ	λ = 1/3, θ = 0°	1/3, ±90°	1/6, 0°	1/6, ±90°	1/10, 0°	1/10, ±90°
For $\tau=0{,}2$						
0	0,113	0,151	0,148	0,157	0,169	0,172
0,01	0,111	0,150	0,146	0,156	0,168	0,170
For $\tau=0{,}4$						
0	0,242	0,327	0,320	0,338	0,367	0,372
0,01	0,239	0,323	0,316	0,334	0,363	0,368

It can also be concluded from the study of the collapse of a bubble between two solid walls in [1, 2] and in the present work that the experimentally observed decrease in the erosion caused by acoustic cavitation with decreasing value of b at R ~ b [3] is a consequence of a decrease in the collapse velocity of cavitation bubbles and of the formation of an annular jet rather than a "cumulative" (linear) jet. This circumstance must be taken into account in choosing optimum technological conditions for ultrasonic processes involving liquids.

NOTATION

r, θ, spherical coordinates; γ, ratio of specific heats of the gas; R, instantaneous radius of the cavitation bubble; t, time; R_{max}, value of R at t = 0; $\lambda = R_{max}/(2b)$; b, distance from the origin to the walls; P_{2m}, Legendre polynomial of index 2m; $\tau = t(p_\infty/\rho)^{1/2}/R_{max}$, dimensionless time; p_∞, pressure in the liquid at an infinite distance from the bubble; ρ, density of the liquid; v, collapse velocity of the cavitation bubble; $\delta = p_{vg}/p_\infty$, gas-content parameter; $p_{vg} = p_v + p_g$, pressure of the vapor–gas mixture in the bubble at t = 0; p_v, partial pressure of the liquid vapor in the bubble; p_g, partial pressure of the gas (air) in the bubble.

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